



PHYS 102 – General Physics II Final Exam

Duration: 120 minutes

Saturday, 17 May 2025; 09:30

1. A cylindrical shell of radius R and of infinite length carries a uniform surface charge σ .

(a) (5 Pts.) Find the magnitude of the electric field inside and outside the shell, if the shell is static.

(b) (6 Pts.) Find the magnitude of the magnetic field inside and outside the cylinder if the cylinder is rotating with constant angular velocity ω about its symmetry axis.

(c) (6 Pts.) Find the magnitude of the electric field inside the cylinder if the cylinder has constant angular acceleration so that $\omega = \alpha t$.

(d) (8 Pts.) Find the difference between the magnetic field magnitudes on the axis and at $r = R/2$ inside the cylinder if the cylinder's angular acceleration is increasing so that $\omega = At^2$.

Solution: (a) Charge distribution is cylindrically symmetric. Apply Gauss's law where the surface is a concentric cylinder with radius r and length ℓ . Charge per unit length on the cylinder is $\lambda = 2\pi R\sigma$.

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \rightarrow (2\pi r\ell)E(r) = \frac{2\pi R\ell\sigma}{\epsilon_0} \rightarrow E(r) = \frac{R\sigma}{\epsilon_0 r}, (r > R), \quad E = 0, (r < R).$$

(b) If the cylinder is rotating, the surface charge creates a current per unit length $J = 2\pi R\sigma/T$, where $T = 2\pi/\omega$ is the period of revolution. The problem is the same as that of an infinite solenoid.

Because of the symmetry of the current distribution, $\vec{B} = 0$ outside and is uniform inside the cylinder. Ampère's law can be applied to find the field inside.

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{enc} \rightarrow B\ell = \mu_0(R\omega\sigma)\ell \rightarrow B_{in} = \mu_0 R\sigma\omega.$$

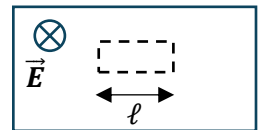
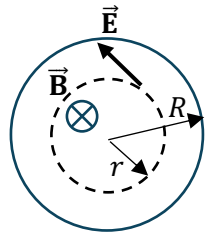
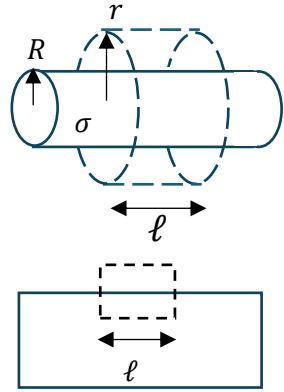
(c) If the cylinder rotates with constant angular acceleration, the current changes with time, and the changing magnetic field $B = \mu_0 R\sigma\alpha t$ creates an induced electric field. The problem is cylindrically symmetric, so applying Faraday's law, we get

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt} \rightarrow (2\pi r)|E(r)| = \frac{d}{dt}[(\mu_0 R\sigma\alpha t)(\pi r^2)] \rightarrow |E(r)| = \frac{1}{2}\mu_0 R\sigma\alpha r, \quad r < R.$$

(d) If the acceleration of the cylinder changes with time, the changing electric field creates a displacement current, thus inducing a magnetic field. Using Ampère-Maxwell law, we find

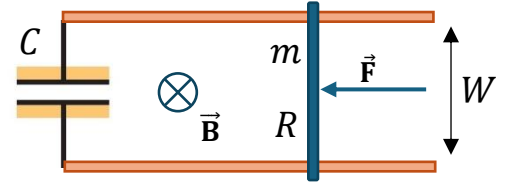
$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}, \quad E(r) = \mu_0 R\sigma\alpha r$$

$$[B(0) - B(R/2)]\ell = \mu_0 \epsilon_0 \frac{d}{dt} \int_0^{R/2} (\mu_0 R\sigma\alpha r')\ell dr' \rightarrow \Delta B(r) = \frac{1}{8}\mu_0^2 \epsilon_0 \sigma \alpha R^3.$$



2. A capacitor with capacitance C is connected to two horizontal, parallel, conducting rails with negligible resistance separated by a distance W . A conducting rod of mass m and resistance R can slide on the rails without friction. A magnetic field of magnitude B exists in the whole region between the rails perpendicular to the plane of the rails. An external force acts on the rod causing it to move with velocity $v(t) = v_0 \sin(\omega t)$ in the direction perpendicular to the rails.

- (a) (8 Pts.) Find the current $i(t)$ induced in the circuit.
 (b) (8 Pts.) Find the average power dissipated by the resistor.
 (c) (9 Pts.) Find the peak value of the external force acting on the rod.



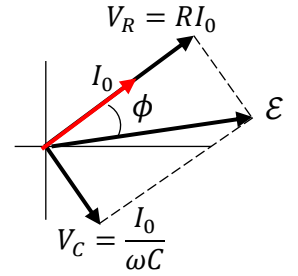
Solution:

(a) Motional emf induced along the length of the moving rod is $\mathcal{E} = BWv_0 \sin(\omega t)$. Therefore, there will be a current induced in the loop containing the capacitor and a resistance of the rod. According to the loop rule $\mathcal{E} + V_R + V_C = 0$, and voltage on the capacitor will lag the current by $\pi/2$. Hence, we have the following phasor diagram:

$$V_C = \frac{q}{C}, \quad V_R = Ri(t) \rightarrow BWv_0 \sin(\omega t) + Ri(t) + \frac{q}{C} = 0$$

$$(BWv_0)^2 = (RI_0)^2 + \left(\frac{I_0}{\omega C}\right)^2 \rightarrow I_0 = \frac{BWv_0\omega C}{\sqrt{1 + (R\omega C)^2}}, \quad \phi = \arctan\left(\frac{1}{R\omega C}\right)$$

$$i(t) = \frac{BWv_0\omega C}{\sqrt{1 + (R\omega C)^2}} \sin(\omega t + \phi)$$



(b)

$$P_{\text{av}} = \frac{1}{2} RI_0^2 \rightarrow P_{\text{av}} = \frac{1}{2} R \left(\frac{B^2 W^2 v_0^2 \omega^2 C^2}{1 + (R\omega C)^2} \right)$$

(c) Since $v(t) = v_0 \sin(\omega t)$, the acceleration of the rod is $a(t) = v_0 \omega \cos(\omega t)$.

$$F_{\text{net}} = F_{\text{ext}} - F_B = ma(t) \rightarrow F_{\text{ext}} = mv_0 \omega \cos(\omega t) + Bi(t)W$$

$$i(t) = I_0 \sin(\omega t + \phi) = I_0 \sin \omega t \cos \phi + I_0 \cos \omega t \sin \phi$$

$$F_{\text{ext}} = (mv_0 \omega + BWI_0 \sin \phi) \cos \omega t + BWI_0 \cos \phi \sin \omega t$$

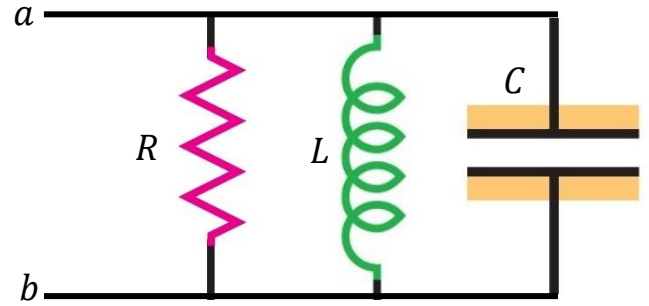
$$|F_{\text{ext}}|_{\text{max}} = \sqrt{m^2 v_0^2 \omega^2 + 2mv_0 \omega BWI_0 \sin \phi + B^2 W^2 I_0^2}, \quad \sin \phi = \frac{1}{\sqrt{1 + (R\omega C)^2}}$$

$$|F_{\text{ext}}|_{\text{max}} = \sqrt{m^2 v_0^2 \omega^2 + \frac{2mB^2 W^2 v_0^2 \omega^2 C}{1 + (R\omega C)^2} + \frac{B^4 W^4 v_0^2 \omega^2 C^2}{1 + (R\omega C)^2}}$$

3. Consider the parallel RLC circuit shown.

(a) (10 Pts.) If a battery with emf $\mathcal{E}$ and internal resistance r is connected to points a and b , what will be the current through the battery at time $t = 0$ and after a very long time?

(b) (15 Pts.) If an AC source $v(t) = V_0 \cos \omega t$ is connected to points a and b , What will be the total current leaving the source as a function of time?



Solution:

(a) At time $t = 0$, the capacitor will behave as a short circuit, while the inductor will have infinite resistance. Therefore points a and b will be shorted. Similarly, after a very long time the inductor will behave as a short circuit, and the capacitor will have infinite resistance. Again, points a and b will be shorted. In this case the only resistance in the circuit will be the internal resistance of the battery. Hence the current I_B through the battery will be

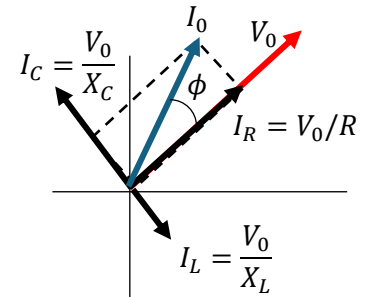
$$I_B = \frac{\mathcal{E}}{r}.$$

(b) If an AC source $v(t) = V_0 \cos \omega t$ is connected to points a and b , voltage on all three circuit elements will be the same, while the currents will satisfy $i(t) = i_R(t) + i_L(t) + i_C(t)$. Current through the resistor will be in phase with the voltage, current through the inductor lags the voltage by $\pi/2$, and current through the capacitor leads the voltage by $\pi/2$. Hence, we have the following phasor diagram:

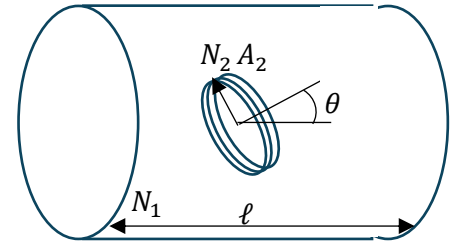
$$I_0 = \sqrt{I_R^2 + (I_C - I_L)^2} = \sqrt{\left(\frac{V_0}{R}\right)^2 + \left(\frac{V_0}{X_C} - \frac{V_0}{X_L}\right)^2}, \quad \tan \phi = \frac{I_C - I_L}{I_R}$$

$$I_0 = V_0 \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}, \quad \phi = \arctan\left(\omega RC - \frac{R}{\omega L}\right)$$

$$i(t) = I_0 \cos(\omega t + \phi)$$



4. (25 Pts.) A small thin coil with N_2 loops, each of area A_2 , is placed inside a long solenoid, near its center. The solenoid has N_1 loops in its length ℓ , and has area A_1 . Determine the mutual inductance as a function of θ , the angle between the normal to the plane of the small coil and the axis of the solenoid.



Solution:

The magnetic field magnitude inside the solenoid is found by using Ampère's Law.

$$B = \mu_0 n I = \frac{\mu_0 N_1 I}{\ell}.$$

Direction of the magnetic field is parallel to the symmetry axis of the solenoid. Therefore, flux of the magnetic field through the thin small coil is

$$\Phi_{2B} = N_2 B A_2 \cos \theta = \frac{\mu_0 N_1 N_2 A_2 I}{\ell} \cos \theta$$

The mutual inductance between the solenoid and the small coil is

$$M = \frac{\Phi_{2B}}{I} = \frac{\mu_0 N_1 N_2 A_2}{\ell} \cos \theta.$$